Determining Hydraulic Ram Pump Feasibility

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ABSTRACT: Hydraulic ram pumps have been used for over 200 years to pump water using only the potential energy of elevated water. Today, the low price and simplicity of the materials and methods required to construct ram pumps make them an excellent option for small and mid-sized water systems in developing countries where elevated water is available. However, ram pumps are risky to implement because they will fail to deliver water if they are not designed correctly. Currently, the evaluation of ram pump designs requires complex computer modeling typically unavailable in developing countries. Existing design documents for users without a specialization in fluid mechanics use vague rules of thumb to simplify the design process. Unfortunately, these rules cannot reliably predict whether a ram pump will deliver water in many cases. This study models the acceleration of fluid in the drive pipe and the intensity of the pressure spike to determine the feasibility of a wide variety of ram pump designs with a higher degree of certainty than previous rules of thumb. The model can be used in a Matlab program that determines if a design will function based on design parameters input by the user. The Matlab method was used along with conservative assumptions to provide design guidelines that can be easily applied for a range of pumping requirements. The model also highlights the importance of the fall to length ratio of the drive pipe, which can be leveraged to improve performance. These findings encourage a wider proliferation of hydraulic ram pumps through more accurate design tools and can reduce the cost of water systems for small developing communities.

KEYWORDS: hydraulic ram pumps, rural water systems, off-grid systems, hydraulic system modelling

1 INTRODUCTION

1.1 Overview

Hydraulic ram pumps are devices capable of using the potential energy of elevated water to lift a fraction of that water to higher elevation. They are 'powered' by the water hammer effect that occurs when their waste valve slams shut due to the force of flowing water. Ram pumps are useful in rural or developing areas because they require no power source, no specialized training to maintain, and can be effectively built and repaired with basic construction methods and widely available pipe fittings. These characteristics are useful because reliable electrical grids, specialized parts, and skilled labor required by externally powered pumps are costly or unavailable in developing countries.

Unfortunately, ram pump systems are difficult to design because their operation and performance is highly dependent on numerous aspects of the site geography and pump design. Commonly used design methods are highly uncertain because they follow vague rules of thumb or use unreliable equations (Young 1997). Numerical analyses using a partial derivative formulation have successfully predicted performance characteristics for a small amount of experimental data; however, the methods developed are difficult to implement for users without a high competency in numerical methods and programming (Filipan, Virag, & Bergant 2003). Several non-dimensional correlations for ram pump performance have been proposed based on experimental data (Fatahi-Alkouhi & Lashkarara 2017; Young 1997). However, these correlations cannot be used to determine the limitations of a design because they do not fully characterize the acceleration of flow in the drive pipe.

The service-learning organization Clemson Engineers for Developing Countries has cancelled multiple ram pumps projects because the site conditions are not robust enough to guarantee a ram pump will function. This uncertainty has likely led to the underuse of ram pumps by humanitarian organizations and developing communities themselves, as both groups are unlikely to invest in water projects that may not deliver sufficient or any water.

This investigation models the ram pump operating cycle with basic fluid mechanics principles to develop a Matlab based method for evaluating ram pump designs. The method provides the design boundaries of feasible ram pump systems with less uncertainty than current design methods. The user does not require any technical background to evaluate the design and can analyze a wide array of possible designs due to the fundamental nature of the model. The Matlab script was used to produce a reference table that provides the minimum site requirements for a pump to be feasible across several common scenarios. Users without access to Matlab can also solve the nonlinear differential equations in the model with a simple Euler approach using a spreadsheet software. The investigation also indicates the importance of the fall height to drive pipe length ratio, which has been absent from previous non-dimensional analyses.

1.2 Definition of Terms

- Fall Height Elevation change from the top of the water source to the waste valve (or water line above the waste valve if it is submerged.)
- Lift Height Elevation change from the internal check valve of the pump to the location water will be delivered.
- Fall to Length Ratio Equal to the fall height divided by the drive pipe length.
- Spike Velocity The average velocity of the water in the drive pipe at the instant the waste valve slams shut.
- Spike Pressure The peak pressure created each time the waste valve slams shut.

1.3 Definition of Mathematical Symbols

- h f fall height (supply head)
- h₁ lift height (delivery head)
- h_{1 max} maximum lift height of a pump

- F force on the waste valve wafer due to the waste flow
- F_a gravitational force accelerating the drive pipe flow
- F _p frictional force on the drive pipe flow from the drive pipe wall
- F _m force acting on the drive pipe flow due to minor losses throughout the drive pipe
- f _d Darcy friction factor
- m mass of water in the drive pipe
- r inner radius of the drive pipe
- r_w radius of the waste valve wafer
- V flow velocity
- V_s spike velocity
- P_s spike pressure
- P _{loss} instantaneous pressure loss per unit length in the drive pipe
- t _s acceleration time required to reach spike velocity
- x distance traveled by the flow
- x _s distance flow must advance each pump cycle to reach the spike velocity
- C $_{\rm w}$ wave velocity in the drive pipe
- B bulk modulus of water
- K Minor loss coefficient
- E Young's modulus of the drive pipe material
- e wall thickness of the drive pipe
- D inner diameter of the drive pipe
- A $_{\rm c}$ cross sectional area of the drive pipe
- 1 drive pipe length
- T $_{\rm w}$ pipe wall shear stress
- ϵ absolute roughness of the drive pipe
- Re Reynold's number
- μ dynamic viscosity of water
- ν kinematic viscosity of water
- Q_s flowrate supplied to pump
- Q_d flowrate delivered by the pump
- Q w waste flow rate of pump
- + Q $_{\rm c}$ instantaneous waste flowrate immediately preceding the pressure spike
- ρ density (water)



Figure 1: Ram pump system schematic

1.4 Ram pump setup and cycle

The typical ram pump installation is illustrated in Figure 1.

The ram pump uses the elevated water source to initiate flow through its drive pipe and waste valve (typically a check valve). The flow accelerates due to the gravitational potential energy of the fall height at a rate determined by the fall to length ratio until the force of the water moving though the waste valve slams it shut. At this point, the water stops suddenly, leading to a pressure spike due to the water hammer effect. This pressure opens the internal valve (also a check valve) and moves a small amount of water through it against the head pressure of the lift height. Concurrently, the pressure spike sends a pressure wave up the drive pipe to the free surface of the water source. Once the wave reaches the free surface, a low-pressure wave propagates back down the drive pipe until it reaches the internal valve. This causes the internal valve to close due to the lift height pressure, and the waste valve to open. Water begins to accelerate out of the waste valve again and the cycle repeats indefinitely (Glover 1994).

1.5 Current design methods

Currently, most ram pump systems are designed by assuming a certain input flowrate and an efficiency of 60% (Rife Hydraulic Engine Manufacturing Co 1985; Smith 2017). These two parameters can be used along with the lift height and fall height to calculate the delivery flowrate using the definition of pump efficiency given in equation 1.

$$\eta = \frac{Q_d h_l}{Q_s h_f} \tag{1}$$

Arbitrarily assuming an efficiency of 60% is problematic because real efficiencies can range from at least 0 to 65% and vary as a function of as many as 11 independent variables (Fatahi-Alkouhi & Lashkarara 2017). Most importantly, this design method leaves users to guess the input flowrate and fall height their pump will require across wide experimental ranges. This guess is critical because the pump will fail if the water source flowrate or fall height is not sufficient to generate a spike pressure capable of opening the internal check valve. This method ignores the impact of low fall to length ratios, which can also cause the pump to fail due to insufficient acceleration of the drive pipe flow.

Other methods use experimental data and dimensionless groups to predict performance criteria. However, these methods do not consider cases where maximum velocity or input flowrate of the water is insufficient for certain lift heights (Fatahi-Alkouhi & Lashkarara 2017; Young 1997). As a result, they cannot be used to reliably determine the feasibility of a design.

2 METHODOLOGY

A model was developed to predict the fall height, fall to length ratio, and drive pipe flowrate required to achieve a certain spike pressure. The model can be applied to any ram pump design with a single waste valve.



Figure 2: Mark Risse ram pump design

A ram pump was constructed from PVC pipe and fittings to conduct experiments from a design by Mark Risse (Smith 2017). A schematic of the design is shown in Figure 2.

The design features a drive pipe with an inner diameter of 34.5 mm and a wall thickness of 7.6 mm. Experiments were conducted using both a brass swing check and weighted poppet-style brass spring check valves for the pump's waste valve, which is shown as item 4 in Figure 2. A PVC poppet-style spring check valve was used for the internal valve which is shown as item 5 in Figure 2. The same nominal size as the drive pipe was used for the internal valve and waste valves. The pump used for testing is shown in the image below, where a wire wheel brush is attached to the waste valve wafter to increase its weight.

Increased lift heights were simulated by partially closing a globe valve on the end of the delivery line. The maximum lift height, corresponding to an efficiency of zero, was measured by closing the globe valve completely. The experimental lift height was determined by reading a pressure gauge at the bottom of the delivery line and translating it to a pressure head. This method allowed experiments to be run with delivery heads as high as 38.7 meters.

The ram pump was tested under five different cases with unique combinations of fall height, waste valve weight, and drive pipe length. For each case where the water velocity was sufficient to close the waste valve, the ram pump was tested at multiple simulated lift heights. The input and output flowrate were measured for each lift height by



Figure 3: The operating test ram pump

recording the time required to fill a container of known volume. The parameters of each test case are shown in Table 1.

| Case Number | Fall Height [m] | Valve Weight [kg] | Drive Pipe Length [m] | Maximum Lift Height [m] | |
|-------------|-----------------|-------------------|--------------------------|----------------------------|--|
| 1 | 3.58 | 0.153 | 14.72 | 33.8 | |
| 2 | 3.58 | 0.286 | 14.72 | 49.3 | |
| 3 | 3.58 | Swing Check | 14.72 | 42.3 | |
| 4 | 1.83 | Swing Check | 11.06 | 42.3 | |
| 5 | 1.83 | 0.286 | 11.06 | 0 | |

Table 1: Experimental test cases

For the fifth case, the maximum water velocity at the waste valve was not enough to close the waste valve regardless of input flowrate. As a result, no water could be delivered regardless of simulated lift height. The pump efficiency for each test was calculated according to Equation 1 by using the simulated lift height, the fall height, and the measured input and output flowrates. These results are shown in Figure 4.

The range of efficiencies reported is between 0 and 57%. The results indicate the efficiency is dependent on several variables including the fall height, lift height, and waste valve properties. The maximum lift height for cases 1-4 along with the complete failure of the pump in case 5 highlight the possibility for ram pumps to fail to deliver water after installation.

The operation of a generalized ram pump was modeled to provide a conservative estimate of the maximum lift height that can be achieved assuming the user can modify the weight of a circular waste valve wafer.

3 EVALUATION

3.1 Estimating spike pressure

To ensure the drive pipe has enough length to fully develop the pressure spike, the drive pipe length should be sized between 150 and 1000 times the diameter of the pipe (Calvert 1958). In a pump with a properly sized drive pipe, the spike pressure reaches its maximum intensity, which can be calculated from Equation 2 (Joukowsky 1904).

$$P_s = V_s \rho C_w \tag{2}$$

where the wave velocity, C_w, can be calculated with Equation 3.

$$C_w = \sqrt{\frac{1}{\rho\left[\frac{1}{B} + \frac{D}{Ee}\right]}} \tag{3}$$



Figure 4: Experimental ram pump efficiencies

To determine how accurate the application of this equation was to experimental ram pumps, three tests were conducted to measure the pressure spike with known waste valve wafer masses. For each test, a theoretical spike velocity was calculated based on the mass of the waste valve wafer. This velocity was used to calculate the theoretical pressure spike. The experimental pressure spike was measured by closing the valve on the pump delivery line to simulate the pump operating at maximum lift height (with no delivery flow). The pressure was read from a standard gauge at the bottom of the delivery line between spikes.

An equation describing the water velocity needed to overcome the mass of the valve wafer was needed to complete this analysis. Since the waste valve closure must be rapid for the pump to function properly, the valve was modelled to close immediately once the gravitational force of its wafer mass was exceeded. The force of the flow on the stationary valve wafer is between that of a jet and a disk experiencing drag in an external flow because there is a limited amount of free area around the valve wafer. Modelling the flow's force as that of a jet was found to predict the spike pressure more accurately than using the drag force equation. For waste valves with a circular wafer the force due to a jet reduces to Equation 4.

| Mass [grams] | Theoretical velocity [m/s] | Theoretical spike [kPa] | Experimental spike [kPa] | Ratio [-] |
|--------------|-------------------------------|----------------------------|-----------------------------|-----------|
| 153 | 1.06 | 399 | 345 | 0.863 |
| 286 | 1.45 | 547 | 483 | 0.883 |
| 211 | 1.24 | 470 | 421 | 0.896 |
| | | | | |

Table 2: Calculated and measured spike pressures

$$F = \rho(\pi r_w^2) V_s^2 \tag{4}$$

The force analysis for swing check valves is more involved because the orientation of the wafer is not perpendicular to the waste flow. It was experimentally determined that the spike velocity for a 1 ¼ inch brass swing check valve oriented vertically was approximately 1.22 m/s. This result was similar to the 1.34 m/s close velocity listed in friction loss tables (PlumbingSupply.com 1995). The spike velocity of a swing check valve is a function of the mass per unit area of the wafer and its angle relative to the flow path. Since these properties change little between valve sizes, approximations of the spike velocity between 1.22 and 1.34 m/s are assumed valid for brass swing check valves of other sizes.

The data from these experiments is shown in the table below, where the theoretical velocity is calculated by solving Equation 4 for spike velocity.

From the experiments, it appears the spike pressure is between 85 and 90% of the theoretical spike pressure. Based on this observation, the model uses Equation 2 and multiplies the product by 0.8 to calculate the spike pressure of a pump. This factor adjusts the calculation to account for the experimental differences and provides a small safety factor. The spike pressure can be divided by the water density and acceleration due to gravity to calculate the maximum lift height of the pump. Accurately predicting the spike velocity is important to predict the waste valve weight and the amount of inflow the pump requires.

3.2 Acceleration of drive pipe flow

The acceleration of flow in the drive pipe must be modelled to predict the amount of water required to complete a pump cycle, the pump's frequency, and the maximum spike velocity attainable. These quantities are used to calculate the lift heights a pump can achieve, and the corresponding supply flowrates required.

Each time a ram pump cycles, the water in the drive pipe begins at a negative velocity from the recoil of the previous cycle (Glover 1994). This negative velocity is generally small in comparison to the spike velocity. The model assumes the flow is stationary at the beginning of each cycle, which simplifies the analysis while yielding an upper bound for the required inflow. If the pump is designed correctly, gravity causes the flow to accelerate until the flow reaches the spike velocity required to close the waste valve. However, if the fall height, or fall to length ratio is insufficient, the flow will stop accelerating before the waste valve closes and the pump will not operate.

The fluid inside the drive pipe is analyzed as a control volume, and the acceleration of the flow is modelled as one-dimensional incompressible flow. These assumptions eliminate relative motion in the fluid, allowing for an analysis based on rigid body translation (Munson, Okiishi & Huebsch 2009). When analyzed as a rigid body, the control volume acceleration can be described by Newton's second law as shown in Equation 5,

$$m\frac{dv}{dt} = F_a - F_p - F_m \tag{5}$$

where m is the mass of the accelerated water in the drive pipe, F_a is the acceleration force due to gravity, F_p is the force associated with pipe friction, and F_m is the force associated with minor losses including the waste valve and any other flow restrictions.

The total acceleration force acting on the control volume is the product of the total hydrostatic pressure available, and the cross-sectional area of the pipe as given in Equation 6.

$$F_a = \rho g h_f A_c \tag{6}$$

The pipe friction force can be expressed as the product of the wall shear stress and the total pipe surface area as given in Equation 7.

$$F_p = T_w \pi D l \tag{7}$$

The shear stress on the pipe wall can be modelled with the same methods used to predict pressure drop in pressurized systems. Pressure drop per unit length is related to the wall shear stress by Equation 8 (Munson, Okiishi & Huebsch 2009).

$$\frac{dp}{dl} = \frac{2T_W}{r} \tag{8}$$

The Darcy-Weisbach equation was used to model pressure drop. It is given in Equation 9,

$$\frac{dp}{dl} = f_d \frac{\rho V^2}{2D_h} \tag{9}$$

where f _d is the Darcy friction factor. While the flow is laminar f _d = 64/Re. In this case the pipe friction force is given by Equation 10.

$$F_p = 8\pi l V \mu \tag{10}$$

For most ram pump applications, the drive pipe flow becomes turbulent soon after acceleration begins. For turbulent flow, f $_{\rm d}$ can be approximated explicitly by Equation 11 (Swamee and Jain 1976).

$$f_d = \frac{0.25}{\left(\log_{10}\left(\frac{\epsilon}{3.7D} + 5.74\left(\frac{\nu}{VD}\right)^{0.9}\right)\right)^2}$$
(11)

Rewriting Equation 7 using Equations 9 and 11, yields an expression for pipe friction force when the flow is turbulent in terms of physical constants and velocity. This expression is given in Equation 12.

$$F_{p} = \frac{\rho V^{2} \pi D l}{32 \left(log_{10} \left(\frac{\epsilon}{3.7D} + 5.74 \left(\frac{\nu}{VD} \right)^{0.9} \right) \right)^{2}}$$
(12)

The force associated with minor losses is modeled with the excess head method. The excess head method assumes the pressure loss across an obstruction is proportional to the square of the velocity. The loss coefficient, or proportionality constant, K, of various bends, valves, and fittings can be found in a wide range of reference materials (Munson, Okiishi & Huebsch 2009). A loss coefficient may also be used to account for the presence of a course filter at the drive pipe inlet if one is used. The force due to minor losses can be expressed as the summation of all the minor losses that would be expected in a pressurized system multiplied by the cross-sectional area of the drive pipe. It is assumed here that any obstructions causing minor losses have the same nominal diameter, and therefore the same one-dimensional velocity, as the drive pipe. This assumption yields Equation 13.

$$F_m = \sum K \rho \frac{v^2}{2} A_c \tag{13}$$

The rigid body acceleration described in Equation 5 can now be written for the general case as shown in equation 14.

$$m\frac{dV}{dt} = \rho g h_f A_c - \frac{1}{8}\pi D l \rho V^2 f_d - \sum K \rho \frac{V^2}{2} A_c$$
(14)

The equation can be simplified by replacing m with the product of fluid density and drive pipe volume, and solving for the acceleration term. The result for the laminar case is shown in Equation 15.

$$\frac{dv}{dt} = g \frac{h_f}{l} - 32 \frac{vV}{D^2} - \frac{\sum KV^2}{2l}$$
(15)

The result for the turbulent case is shown in Equation 16.

$$\frac{dV}{dt} = g \frac{h_f}{l} - \frac{V^2}{8D \left(log_{10} \left(\frac{\epsilon}{3.7D} + 5.74 \left(\frac{\nu}{VD} \right)^{0.9} \right) \right)^2} - \frac{\sum KV^2}{2l}$$
(16)

Equations 15 and 16 indicate the flow will stop accelerating (i.e., dV/dt = 0) at a velocity determined by the fall to length ratio, the characteristics of the drive pipe, and the total loss coefficient of the system. This implies that for a system where the effective weight of the waste valve can be adjusted, the fall to length ratio limits the spike velocity even for designs with large fall heights. This insight is largely absent in the literature.

3.3 Direct applications of the acceleration model

Matlab's 'ode45' a fifth order Runge Kutta ordinary differential equation solver is used to solve equation 14 for V(t) and its integral x(t) where the initial value of both variables is equal to zero. The equation can also be solved using a simple Euler technique in a spreadsheet software. The solutions can be used to estimate how much time, t_s , it takes the water to reach a certain spike velocity, V_s . This time can be used to calculate the maximum number of times a pump can cycle each minute. The solutions can also be used to calculate x_s , which represents the distance the flow must advance during each cycle and can be multiplied by the area of the drive pipe to obtain the waste flow volume required for each cycle.

To predict the maximum waste flowrate for design purposes, the negative velocity in the drive pipe after each cycle was neglected. The maximum frequency can be calculated by accounting for the time it takes for the flow to reach the spike velocity, and the time it takes the pressure wave to travel up and down the drive pipe. The maximum frequency can be found from Equation 17.

$$f = \frac{1}{t_s + \frac{2l}{c_w}} \tag{17}$$

Frequencies measured during testing were compared to the maximum frequency predicted by the model. The comparison is shown in Figure 5 where the frequency is displayed in cycles per minute.

The actual frequency of the pump during experimentation was found to be between 60% and 90% of the maximum value. This percentage is influenced by the recoil velocity magnitude, which is influenced by the lift height, fall height, pressure spike, and fall to length ratio of the system. The maximum waste flowrate required for the pump can be calculated by taking the product of the maximum frequency, the cross-sectional area of the drive pipe, and the distance the flow must advance for each cycle, x s.



Figure 5: Experimental and modelled maximum frequency

To determine the maximum input flowrate required, the delivery flowrate must be known or assumed. The delivery flowrate can be expressed by rearranging Equation 1 as shown in Equation 18.

$$Q_d = \frac{\eta h_l Q_w}{(h_l + \eta h_f)} \tag{18}$$

Setting η equal to unity provides the maximum possible delivery flowrate, which can be added to the maximum possible waste flowrate to determine the maximum input flowrate required. It should be noted that the actual delivery flowrate cannot be calculated by the model, since no attempt was made to model the real efficiency.

The model was run over the same conditions as the experiments. The amount of waste water required for the experiments was compared to the maximum amount of waste water the model predicted. This comparison is displayed in the Figure 6.

The comparison indicates that the model is reasonably accurate. This uncertainty can be mitigated by using a small safety factor for the flowrate. The safety factor should also account for seasonal changes in the supply flowrate.



Figure 6: Experimental comparison of waste water flowrate

Designers can use the projected maximum flowrate to determine if their water source has a sufficient flowrate.

3.4 Estimating minimum fall height required

The maximum spike velocity of a ram pump can also be limited by the available fall height. To determine the amount of fall height required for a certain spike velocity, conservation of energy was applied to the flow in the drive pipe using Bernoulli's equation. As the flow in the drive pipe accelerates, the gravitational potential energy of the water is lost to kinetic energy and friction. The pressure loss that occurs during the acceleration of the flow to the spike velocity depends on the behavior of the velocity and the distance the flow translates during a cycle. The amount of gravitational potential energy per unit volume required to close the waste valve is shown in Equation 19 where V is a function of time which can be calculated from integrating Equation 14.

$$\rho g h_f = \frac{1}{2} \rho V(t)^2 + \int_0^{x_s} P_{loss} (V(t)) dx$$
(19)

The function P $_{loss}$ is the pressure drop per unit length of the drive pipe, which includes the pressure loss due to pipe friction (major loss) and waste valve friction (minor loss). P $_{loss}$ can be calculated with Equation 20.

$$P_{loss} = \frac{1}{2l} K \rho V(t)^2 + f_d \frac{\rho V(t)^2}{2D_h}$$
(20)

The integral in Equation 19 is calculated according to the trapezoidal rule, where an average value of P _{loss} during a time interval is determined from the numerical solution for V(t). This value is then multiplied by the change in x(t) over the time interval which can be obtained by integrating Equation 14 twice. These pressure drops are continually summed until x(t) reaches x_s at which point the integration is complete.

Before the waste valve reopens to begin the next cycle, the low-pressure wave from the water source has propagated back down the drive pipe. Since this occurs very quickly in comparison to the acceleration of the flow, the drive pipe flow will begin accelerating with all of the potential energy of the fall height. This implies that the friction head loss does not accumulate each cycle.

This model is implemented in the Matlab script in the appendix. The script can be used to estimate the maximum spike pressure a pump can achieve as a function of drive pipe (length, diameter, material, wall thickness, average angle, bends), fall height, and the available supply flowrate.

4 DELIVERY FLOW OBSERVATIONS

Existing models of the delivery flowrate were compared against the experimental data obtained in this study to determine how accurately they predicted delivery flow for the experimental design. The first model is from Young (1997) and uses the peak instantaneous waste flowrate, Q c, as shown in Equation 21 to directly predict delivery flow.

$$\frac{Q_d h_l}{Q_c h_f} = 0.27 \pm 0.05 \tag{21}$$

When the results of this model were compared to experimental data, the model was found to be inaccurate as the ratio of lift height to fall height increased. The percent error of the experimental data is plotted in the Figure 7 with the lift to fall height ratio on the x-axis.

The figure indicates this model significantly underpredicts delivery flow as the lift to fall height ratio increases.

The data were also compared with an efficiency equation of a Jundi-Shapur University of Technology study derived from experimental results. The relationship is shown in Equation 22 (Fatahi-Alkouhi & Lashkarara 2017).

$$\eta = -0.2688 + \left(\frac{l}{D}\right)^{-0.0479} - 0.4763 \left(\frac{h_l}{h_{lmax}}\right)^{1.2507}$$
(22)

This equation was found to predict pump efficiency accurately for the tests with a fall to lift ratio of 0.17. However, the model underpredicted the efficiency for tests with a fall to length ratio of 0.24. The comparison is shown in Figure 8.

The comparison indicates the model predicted only a slight difference in efficiency for the change in l/D, while the experimental data indicates a much larger difference. This is likely because the model does not directly account for the influence of the fall to length ratio. A steeper fall to length ratio will likely decrease recoil velocity because there is more resistance to backwards flow. This results in



Figure 7: Experimental error in comparison to Young's model



Figure 8: Experimental comparison of efficiency to Jundi-Shapur University model

more of the pressurized flow being delivered during each pressure spike which raises pump efficiency. Additionally, the analysis in Section 3 shows higher fall to length ratios will increase the acceleration of the flow, leading to a lower waste flowrate and higher efficiency.

Since existing correlations could not predict the delivery flow of this design reliably, the model developed in this paper does not predict the delivery flow of potential designs.

5 PRODUCING REFERENCE TABLES FROM MODEL

The acceleration model was conservatively applied to several example scenarios to determine the minimum site characteristics required to pump water for ram pumps with waste valve weights that can be adjusted. In this application it is assumed a site must produce a spike pressure 1.3 times greater than the lift height pressure to be considered feasible. This ratio was chosen based on experimental results that show delivery flow decreases sharply after this ratio. The ratio is similar to the 1.67 ratio recommended by Young (1997) when it is divided by the 0.8 factor used to determine the difference between theoretical and experimental pressure spike (Young 1997). These solutions also assume the drive pipe contains no tight bends, a spring check valve is used for the waste valve (with an assumed loss coefficient of 10), and the l/D ratio is at least 300. Systems that do not meet these criteria need to be evaluated with the Matlab script in the appendix or other numerical methods. To simplify the use of the figure, the results are displayed in both Metric and English units.

Figures 9 and 10 list the minimum site characteristics required to deliver water to different ranges of lift height using Metric and English units respectively. To evaluate a site, one should start by the finding the applicable lift height range. Then, compare the site's attributes with the minimum fall to length ratio (h_f/l) , fall height (h_f) , and

supply flowrate (Q_s) required for the types of drive pipes being considered for the design. If the site meets or exceeds all three of these requirements, a ram pump is feasible for the relevant drive pipe type. For some cases the angle and fall height boxes contain two values separated by a " symbol. In these cases, there are two sperate minimums for the fall to length ratio and fall height. For example, Figure 9 indicates a pump with a 26 mm inner diameter and 3.4 mm wall thickness made of PVC with a lift height between 30 and 46 meters and a fall to length ratio of 0.17 would require 6.71 meters of fall height, but if the pump had a fall to length ratio of 0.26, only 4.27 meters of fall height would be required. If the site exceeds some requirements but falls short in others the design needs to be studied using the Matlab script (included in the appendix) to determine feasibility.

As any pump approaches the minimum requirements for its lift height, the efficiency and therefore the flow rate it will deliver decreases exponentially as shown in Figure 8. The delivery flow of a system will largely depend on the system's drive pipe (diameter, length, and average angle) and fall height. Sites that require low values for those characteristics, such as steel pipes for low delivery heights, will deliver a smaller fraction of their inflow. The arrangement of the site should seek to exceed the minimum requirements to deliver more water. Stand pipe systems can often be used to increase the fall to length ratio if necessary (Rife Hydraulic Engine Manufacturing Co 1985). For a design with an adjustable waste valve to operate correctly, its valve wafer must have the correct amount of weight to prevent the valve from closing until the flow reaches the desired spike velocity. Weight can be added to most spring check valves by disassembling them and attaching mass to the stem before reassembly. If the site characteristics surpass the minimum requirements, more mass can be added to the waste valve wafer to deliver more water. However, adding additional mass will raise the amount of supply flow required, and may raise the fall height, and average drive pipe angle required. The weight required for the maximum spike velocity can also be estimated with the Matlab script or with Equation 4. Figures 11 and 12 list the aggregate mass of the wafer and additional weight in required in grams and ounces respectively for the same scenarios listed in Figures 9 and 10.

6 CONCLUSION

The methods presented allow for the feasibility of a hydraulic ram pump project to be determined by a user without specialized skills with more accuracy than was previously possible. The characterization of the drive pipe acceleration and pressure spike allows users to predict the maximum lift height of most ram pump designs more accurately than previous design methods. While ram pumps will deliver water to lift heights below their maximum, comparisons with experimental data indicated current

| | Pipe Material | | PVC | | | Steel | | |
|---|---------------------|-------------|-------------|--------------|-------------|-------------|-------------|--|
| | Inner Diameter [mm] | 26 | 52 | 77 | 26 | 52 | 77 | |
| | Wall Thickness [mm] | 3.4 | 3.9 | 5.5 | 3.4 | 3.9 | 5.5 | |
| Lift Height [m] | Requirements | | | | | | | |
| | hf/l [-] | 0.13 | 0.09 0.13 | 0.09 0.13 | 0.05 | 0.05 | 0.05 | |
| 23-30 | hf [m] | 2.14 | 4.27 2.74 | 3.96 2.44 | 1.52 | 0.92 | 0.92 | |
| | Q [l/min] | 19.0 | 71.9 | 140.0 | 11.4 | 34.1 | 72.0 | |
| | hf/l [-] | 0.17 0.26 | 0.26 | 0.17 0.26 | 0.09 0.13 | 0.09 | 0.05 0.09 | |
| 30-46 | hf [m] | 6.71 4.27 | 7.01 | 11.28 6.10 | 2.44 1.22 | 1.53 | 3.05 1.22 | |
| | Q [l/min] | 26.5 | 102.2 | 234.7 | 15.1 | 49.2 | 113.6 | |
| | hf/l [-] | 0.34 | 0.26 | 0.26 | 0.13 0.17 | 0.09 0.12 | 0.09 0.13 | |
| 46-61 | hf [m] | 7.62 | 11.59 | 10.67 | 4.57 3.05 | 3.96 2.44 | 3.35 1.83 | |
| | Q [l/min] | 32.2 | 257.4 | 265.0 | 22.7 | 72.0 | 143.8 | |
| | hf/l [-] | 0.50 | Х | Х | 0.26 | 0.13 0.17 | 0.13 0.17 | |
| 61-76 | hf [m] | 12.20 | Х | Х | 4.57 | 4.88 3.66 | 4.88 3.05 | |
| | Q [l/min] | 37.9 | Х | Х | 26.5 | 87.1 | 181.7 | |
| | | | | | | | | |
| May require thicker piping to handle pressure | | | ١ | Not Feasible | | | | |

Figure 9: Minimum site characteristics required for various ram pump configurations in Metric units

| | | PVC Schedule 40 | | | Steel Schedule 40 | | | |
|--|--------------|-----------------|--|-----------|-------------------|-----------|-----------|--|
| Life Linishe [fe1] | Demuinemente | 1 | | 2 : | 1 : | 2 : | 2 | |
| LITT Height [TT] | Requirements | 1 IN | 2 in | 3 IN | 1 IN | 2 in | 3 IN | |
| | | | N. N | | | | | |
| | hf/l [-] | 0.13 | 0.09 0.13 | 0.09 0.13 | 0.05 | 0.05 | 0.05 | |
| 75-100 | hf [ft] | 7 | 14 9 | 13 8 | 5 | 3 | 3 | |
| | Q [gal/min] | 5 | 19 | 37 | 3 | 9 | 19 | |
| | hf/l [-] | 0.17 0.26 | 15 | 0.17 0.26 | 0.09 0.13 | 0.09 | 0.05 0.09 | |
| 100-150 | Hf [ft] | 22 14 | 23 | 37 20 | 8 4 | 5 | 10 4 | |
| | Q [gal/min] | 7 | 27 | 62 | 4 | 13 | 30 | |
| | hf/l [-] | 0.34 | 0.26 | 0.26 | 0.13 0.17 | 0.09 0.12 | 0.09 0.13 | |
| 150-200 | hf [ft] | 25 | 38 | 35 | 15 10 | 13 8 | 11 6 | |
| | Q [gal/min] | 8.5 | 68 | 70 | 6 | 19 | 38 | |
| | hf/l [-] | 0.50 | Х | Х | 0.26 | 0.13 0.17 | 0.13 0.17 | |
| 200-250 | hf [ft] | 40 | Х | Х | 15 | 16 12 | 16 10 | |
| | Q [gal/min] | 10 | Х | Х | 7 | 23 | 48 | |
| | | | | | | | | |
| May require Schedule 80 piping to handle pressure | | | Not Feasible | | | | | |

Figure 10: Minimum site characteristics required for various ram pump configurations in English units

| Pipe Material | PVC | | | Steel | | |
|---------------------|------------------------|------|------|-------|-----|------|
| Inner Diameter [mm] | 26 | 52 | 77 | 26 | 52 | 77 |
| Wall Thickness [mm] | 3.4 3.9 5.5 | | | 3.4 | 3.9 | 5.5 |
| Lift Height [m] | Waste valve weight [g] | | | | | |
| 23-30 | 110 | 600 | 1350 | 32 | 130 | 292 |
| 30-46 | 220 | 1600 | 3750 | 73 | 292 | 658 |
| 46-61 | 375 | 2500 | 5200 | 135 | 520 | 1170 |
| 61-76 | 600 | Х | Х | 203 | 812 | 1827 |

Figure 11: Minimum mass of waste valve wafer required in metric units (grams)

| Lift | PVC Schedule 40 | | | Steel Schedule 40 | | | | | |
|---------|-----------------|-------------------------|------|-------------------|------|------|--|--|--|
| Height | 1 in | 2 in | 3 in | 1 in | 2 in | 3 in | | | |
| [ft] | | Waste valve weight [oz] | | | | | | | |
| 75-100 | 4 | 21 | 48 | 1 | 5 | 10 | | | |
| 100-150 | 8 | 56 | 132 | 3 | 10 | 23 | | | |
| 150-200 | 13 | 88 | 183 | 5 | 18 | 41 | | | |
| 200-250 | 21 | Х | Х | 7 | 29 | 64 | | | |

Figure 12: Minimum mass of waste valve wafer required in English units (ounces)

models for predicting delivery flow are inaccurate in some cases.

Users can make more informed project decisions by using the provided reference scenarios or evaluating specific designs in the numerical model. This information makes ram pump projects less risky investments for those working in developing communities and should lead to a wider proliferation of ram pumps. The model can be used by those designing or troubleshooting ram pump systems to better understand how different variables effect pump performance. The model also highlights the importance of the fall to length ratio for pump feasibility and efficiency.

While the experimental data supported the findings of the model, the data were only recorded over a limited range of design scenarios. Further testing should be conducted across pumps with various design characteristics to evaluate the accuracy of the model across a wider range of scenarios. Additionally, more research is needed to accurately predict the amount of water a pump can deliver each cycle. A dimensional analysis that includes the fall to length ratio as a parameter is a good candidate. Accurately predicting the amount of delivery flow would allow larger ram pump systems to be designed and implemented with a high degree of confidence, which would likely increase the affordability and reliability of water supply systems in the developing world.

7 **REFERENCES**

Calvert, N. (1958) 'Drive pipe of a hydraulic ram', *The Engineer*, Harmsworth Press, London, England, 26 December, 206 (5370), p. 1001

Fatahi-Alkouhi, Reza & Lashkarara, Babak. (2017) 'Experimental evaluation of effective parameters on characteristic curves of hydraulic ram-pumps' *Scientia* Iranica. 26. 10.24200/sci.2017.4597.

Filipan, V., Virag, Z. & Bergant, A. (2003) 'Mathematical modelling of a hydraulic ram pump system' *Journal of Mechanical Engineering* 49(3):137-149

'Friction Loss Tables', (1995) PlumbingSupply.com. https:// www.plumbingsupply.com/ed-frictionlosses.html, Accessed 6 November, 2022

Glover, Peter B.M. (1994) 'Computer simulation and analysis methods in the development of the hydraulic ram pump', University of Warwick Department of Engineering

Joukowsky J.N. (1904) 'Water hammer', Proceedings of the American Waterworks Association, Vol 24 pp 341-424.

Munson, Okiishi and Huebsch (2009) Fundamentals of fluid mechanics 6th ed. 978-0470-26284-9, John Wiley & Sons, Inc

'Rife hydraulic water rams', (1985) Rife Hydraulic Engine Manufacturing Co.

Smith, W. B. (2017) 'Home-made hydraulic ram pump for livestock water', Clemson University Extension Services

Swamee, P. K., and Jain, A. K. (1976) 'Explicit equations for pipe-flow problems' J. *Hydr. Div.*, ASCE, 102(5), 657-664.

Young, B. (1997) 'Design of homologous ram pumps', J. *Fluids Eng.* June 1997, 119(2): 360-365

8 APPENDICIES

8.1 Matlab Hydraulic Ram Pump Design Evaluator

The code below can be used to determine the minimum and maximum operating characteristics of a specific ram pump design. If the model determines the ram pump design is not feasible, the code will output whether the fall height or the amount of supply flow is the limiting factor on pump performance. The additional assumptions used in the code are listed in the first block of comments.

```
% Ram Pump Design Evaluator
% Max Pawlick 8/29/21
% Assumes density of water is 1000kg/m^3 and kinematic viscosity is 0.0000011384; % m^2/s
% Assumes acceleration due to gravity is 9.8 m/s^2
% Assumes the bulk modulus of water is 2.19*10^9 pa
%% Collect Data From User
promptyUnit = 'What unit system is preferred [Metric or English] \n';
%determine unit system for data entry
Unit=input(promptyUnit,'s');
if strcmpi(Unit,'Metric')
% collect data in metric units
promptyM = 'What is the Youngs Modulus of the drive pipe material? [pascals] \n';
promptWThick = 'What is the wall thickness of the drive pipe? [mm] \n';
promptARough = 'What is the absolute roughness of the drive pipe material? [mm] \n';
promptDH = 'What is the inner diameter of the drive pipe? [mm] n';
promptDPipeL = 'What is the length of the drive pipe? [m] \n';
promptFH = 'How much fall height is available? [m] \n';
promptQin = 'What is the minimum available flowrate of the water source? [liters/minute] \n';
promptLH = 'How much lift height does the pump need to overcome? [m] \n';
promptKval = 'What is the estimated total K-Value of the bends in the drive pipe and the waste valve?
(generally between 10 and 15) [-] \n';
yMod=input(promptyM);
wThick=input(promptWThick)/1000; % mm to m
e=input(promptARough)/1000; % mm to m
Dh=input(promptDH)/1000; % mm to m
dpLength=input(promptDPipeL);
supplyFlow=input(promptQin); %kept in liters per minute
liftHeight=input(promptLH);
fallHeight=input(promptFH);
kVal=input(promptKval);
elseif strcmpi(Unit,'English')
promptyM = 'What is the Youngs Modulus of the drive pipe material? [psi] \n';
promptWThick = 'What is the wall thickness of the drive pipe? [inches] \n';
promptARough = 'What is the absolute roughness of the drive pipe material? [inches] \n';
promptDH = 'What is the inner diameter of the drive pipe? [inches] \n';
promptDPipeL = 'What is the length of the drive pipe? [feet] \n';
promptFH = 'How much fall height is available? [feet] \n';
promptQin = 'What is the minimum available flowrate of the water source? [Gallons/minute] \n';
promptLH = 'How much lift height does the pump need to overcome? [feet] \n';
promptKval = 'What is the estimated total K-Value of the bends in the drive pipe and the waste valve?
(generally, between 10 and 15) [-] \n';
```

```
yMod=input(promptyM)*6894.76; % psi to pa
wThick=input(promptWThick)*2.54/100; % inches to meters
```

```
e=input(promptARough)*2.54/100; % inches to meters
Dh=input(promptDH)*2.54/100; % inches to meters
dpLength=input(promptDPipeL)/3.28; % feet to meters
supplyFlow=input(promptQin)*3.785; % gallons to liters
liftHeight=input(promptLH)/3.28; % feet to meters
fallHeight=input(promptFH)/3.28; % feet to meters
kVal=input(promptKval);
```

else

error('Did not recognize input for unit system, enter "Metric" or "English"')
end

g=9.8; %acceleration due to gravity

row=1000; %density of water

vK= 0.0000011384; % kinematic viscosity m^2/s assuming water temp of 15 degrees Celsius

bMW=2.19*10^9; % The bulk modulus of water

%% Complete preliminary calculations

hOverL=fallHeight/dpLength;

deliveryPressure=row*g*liftHeight;

minSpikeP=deliveryPressure*1.3;

waveVel=(1/(row*((1/(bMW))+(Dh/(yMod*wThick)))))^0.5;

minSpikeVel=minSpikeP/(row*waveVel*0.8);

aveV=minSpikeVel*0.7; % assuming the average drive pipe velocity while the flow accelerates is 70%
Re=aveV*Dh/vK; % Average Reynolds number while the flow accelerates. Used to estimate the average Hazen
Williams friction coefficient.
c=(log10((e/(3.7*Dh))+(5.75/(Re^0.9)))/(-0.0432*Dh^0.0093*(Re*vK)^0.074))^1.08; %Calculates the average

Hazen Williams coefficient

Fp1=e/(3.7*Dh); % Fp (1,2,and 3)are used to calculate the resistance of the pipe during turbulent flow
Fp2=5.74*(vK/Dh)^(0.9);

Fp3=1/(2*Dh);

FpL=64*vK/(2*Dh^2); %Represents the resistance of the pipe during laminar flow
Fv=(kVal)/(2*dpLength); %Represents the resistance of the bends and waste valve

```
%% Set up and solve the equations of drive pipe flow
tspan = linspace(0,7,200); % This assumes the spike velocity will be reached in 7 seconds
% tspan may need to be extended for drive pipes larger than 3 inches with
% low fall height to length ratios
tspanL=linspace(0,2,100); % Assumes the flow will transition to turbulent within 2 seconds
initialvaluesL=[0,0]; %Assumes the flow is stationary at the start of each cycle
[tL,xL]=ode45(@(tL,xL) fL(tL,xL,h0verL,FpL,Fv),tspan,initialvaluesL); %Solves for flow translation and
velocity during laminar flow
ReL=xL(:,2)*Dh/vK; %Calculates the Reynolds number for the solution.
turbLoc=max(find(ReL>3000,1)); %Determines the point where the flow transitions to turbulent
initialvalues=[xL(turbLoc,1),xL(turbLoc,2)]; %Defines the initial values from the laminar solutions at
the point of turbulence
[t,x]=ode45(@(t,x) f(t,x,h0verL,Fp1,Fp2,Fp3,Fv),tspan,initialvalues); %Solves for the velocity and
distance traveled by the flow over time
t=t+tL(turbLoc); %Adds the time it took the flow to become turbulent to the turbulent time vector
aLength=size(t);
depth=aLength(1);
```

%% Calculate the amount of fall height needed for various spike velocities

hlPressure=zeros(depth,1);

for hwIndex=2:depth

numericVel=(x(hwIndex-1,2)+x(hwIndex,2))/2; %Calculates the average velocity over the time interval deltaX=x(hwIndex,1)-x(hwIndex-1,1); %Calculates the distance traveled during the time interval numRe=Dh*numericVel/vK; %Calculates the average Reynolds number during the time interval

```
if numRe>3000 %Turbulent flow
```

f_darcy(hwIndex,1)=0.25/(log10(Fp1+Fp2/(numericVel^(0.9))))^2; %Swamee-Jain equation for Darcy
friciton factor

```
else %Laminar flow
```

f_darcy(hwIndex,1)=(64/numRe); %Darcy Weisbach equation

```
end
```

hlPressure(hwIndex,1)=f_darcy(hwIndex,1)*row/2*numericVel^2/Dh*deltaX; %Darcy-Weisbach equation for
pressure drop

% In the line below, to account for waste valve loss row*g on right hand side used to convert meters head to pascals

```
hlPressure(hwIndex,1)=hlPressure(hwIndex,1)+(kVal*numericVel^2)/(2*g)*(row*g);
```

%to account for head loss from previous increments

```
hlPressure(hwIndex,1)=hlPressure(hwIndex,1)+hlPressure(hwIndex-1,1);
```

end

velocityPressure=(row/2)*x(:,2).^2; %Represents the velocity pressure for each possible velocity
totalHeadRequired=hlPressure+velocityPressure; % Adds velocity pressure to frictional losses to determine
total head required

%% Calculating Possible Performance data

frequency=60./(t+(dpLength/waveVel)*2); %Calculates the maximum amount of times the pump will cycle per minute

wasteVolumePerCycle=x(:,1)*(pi/4*Dh^2); %Converts the linear distance traveled by the flow to volume
wasteVolumePerMinute=wasteVolumePerCycle.*frequency;

wasteVLPM=wasteVolumePerMinute*1000; %converts from cubic meters to liters

litersIn=wasteVLPM/(1-(fallHeight/liftHeight)); %This adds the maximum possible delivery flow to
determine the total flowrate required

%% Calculating Feasible Performance Data

possibleVelsLogic1=max(find(litersIn<supplyFlow)); %Determines if there is enough supply flow for each
possible spike velocity</pre>

possibleVelsLogic2=max(find(totalHeadRequired<(fallHeight*row*g)));%Determines if there is enough head
pressure for each possible velocity</pre>

if possibleVelsLogic1<possibleVelsLogic2 %Determines whether head pressure or supply flow is limiting
the spike velocity</pre>

```
velLoc=possibleVelsLogic1;
```

```
diaLimiting="supply flowrate. \n";
```

else

```
velLoc=possibleVelsLogic2;
```

diaLimiting="fall height. \n";

```
end
```

posibleVels=x(1:velLoc,2); %Determines what spike velocities are possible for the system
maxPV=max(posibleVels); % Determines the maximum possible spike velocity

if minSpikeVel>maxPV %Determines if the maximum possible velocity is greater than the velocity required to pump water

outDiaOne=fprintf("Not feasible. The limiting factor of the system is " + diaLimiting);

else

```
% calculates the mass of the valve wafer required to cause the minimum spike velocity
valveMinMass=(row*(pi/4)*Dh^2*minSpikeVel^2)/g;
valveMaxMass=(row*(pi/4)*Dh^2*maxPV^2)/g;
```

%calculates the min and max spike pressure the pump will have to handle in pa

```
spikePressureMin=waveVel*row*minSpikeVel;
```

spikePressureMax=waveVel*row*maxPV;

```
%% Generating Output Report
if strcmpi(Unit,'Metric')
```

```
outDiaOne=fprintf("Feasible \n The pump can operate between spike velocities of " + num2str(minSpikeVel)
+ " and " +num2str(maxPV) + " meters/second. ");
     outDiaTwo=fprintf("\n This will produce spike pressures the pump needs to absorb between " +
num2str(spikePressureMin) + " and " +num2str(spikePressureMax)+ " pa. " );
  outDiaThree=fprintf("\n A circular waste valve wafer would need a mass between " + num2str(valveMinMass)
+ " and " +num2str(valveMaxMass)+ " kilograms to produce this range of spike velocities. \n");
    outDiaFour=fprintf(" Increasing the spike velocity, pressure, and valve weight will increase the
amount of flow delivered, but may cause strain on the pump materials. \n");
    outDiaFive=fprintf(" The power of the system is limited by the available " + diaLimiting);
else
    minSpikeVel_E=minSpikeVel*3.28; % meters to feet
    maxPV_E=maxPV*3.28; % meters to feet
    spikePressureMin_E=spikePressureMin/6894.76; % pa to psi
    spikePressureMax_E=spikePressureMax/6894.76; % pa to psi
    valveMinMass_E=valveMinMass*2.205; % kg to lb
    valveMaxMass_E=valveMaxMass*2.205; % kg to lb
  outDiaOne=fprintf("Feasible \n The pump can operate between spike velocities of " + num2str(minSpikeVel_E)
+ " and " +num2str(maxPV_E) + " feet/second. ");
     outDiaTwo=fprintf("\n This will produce spike pressures the pump needs to absorb between " +
num2str(spikePressureMin_E) + " and " +num2str(spikePressureMax_E)+ " psi. " );
  outDiaThree=fprintf("\n A circular waste valve wafer would need a mass between " + num2str(valveMinMass_E)
+ " and " +num2str(valveMaxMass_E)+ " pounds to produce this range of spike velocities. \n");
    outDiaFour=fprintf(" Increasing the spike velocity, pressure, and valve weight will increase the
amount of flow delivered, but may cause strain on the pump materials. \n");
    outDiaFive=fprintf(" The power of the system is limited by the available " + diaLimiting);
end
end
%% Differential Equation to calculate laminar flow acceleration
function rkL=fL(tL,xL,h0verLF,FpFL,FvFL)
            q=9.8;
            rkL=[xL(2);g*h0verLF-FpFL*(xL(2))-FvFL*(xL(2))^2];
       end
%% Differential Equation to calculate turbulent flow acceleration
  function rk=f(t,x,h0verLF,Fpf1,Fpf2,Fpf3,FvF)
            g=9.8;
            rk=[x(2);g*h0verLF-0.25/((log10(Fpf1+Fpf2/(x(2)^0.9))))^2*Fpf3*x(2)^2-FvF*(x(2))^2];
        end
```